VERIFICATION OF A MODEL FOR ESTIMATING MAXIMUM DEPTH OF CONVECTIVE MIXING

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Submitted in partial fulfillment of
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ABSTRACT

A method of estimating the depth of homogeneous mixing in the upper layer of a two-layer ocean is discussed. The method applies during the period of general heat loss from the ocean surface and is based upon convective processes occurring within the homogeneous layer.

The model used is one developed by N. P. Bulgakov; density and salinity distribution with depth are plotted from the surface through the halocline. The density scale is oriented such that the maximum density for any given salinity is read directly. One can thus represent the maximum density obtainable (for a given salinity at freezing temperatures) by the salinity plot. By also plotting the initial density distribution with depth, and the average salinity attained in a homogeneous layer mixed to the plotted depth, one can determine the maximum depth of convective mixing from the intersection of these curves.

The model was tested using data taken during the winter months over a five year period at Ocean Weather Station "P" located in the northeastern Pacific Ocean.

The conclusions reached show that the model has two important features; first, it clearly demonstrates the important factors causing convective processes to occur; and second, it can be useful in separation of effects of the convective process from those of other mixing or non-mixing processes in the upper ocean layer.

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1. Background.

Knowledge of the vertical thermal structure of the oceans is a prime determinant of successful anti-submarine warfare. Maximum efficiency in the use of acoustic detection devices in the oceans can be achieved only through knowledge of its thermal structure. In particular, the ability to predict the depth of homogeneous mixing in the upper layer would greatly enhance the effectiveness of mobile sonar platforms. Toward this goal, investigation needs to be made concerning processes which help determine the ocean-thermal-structure distribution. In certain geographical areas during particular seasons the process of convective mixing is predominant in the upper layers of the ocean.

This paper reports tests of a model for estimating limits of homogeneous mixing in the ocean due to convective mixing during the cooling season of the year.

2. Introduction.

The mixed-layer depth is defined as the depth to which the upper layer of the ocean has been mixed homogeneously. There are several causes for vertical mixing in the oceans. Basically they can be sub-divided into two categories

1) mixing due to convective turbulence, and
2) mixing due to frictional turbulence caused by wave action and currents.

In addition there are two non-mixing types of fluctuations of the mixed-layer depth which are associated with

1) internal waves, and
2) the vertical convergence-divergence caused by changes in the overlying atmospheric pressure field, or by convergence-divergence in the surface current field.

The possible influence of internal waves and convergence-divergence effects on the mixed-layer depth were not evaluated nor accounted for in this study.

Convective mixing is due to vertical instability. In the cooling season there is a net loss of heat from the ocean surface caused by upward transfer of sensible and latent heat. The subsequent cooling of the water increases the density of the surface layer, thereby decreasing the vertical stability of the water column, or making it

unstable. When instability is caused by cooling, convective mixing occurs down to a depth where the density is equivalent to the increased density of the surface layer. This process occurs until there is no longer a net radiative loss to the atmosphere or until the freezing temperature is reached.

High rates of sensible heat loss to the atmosphere from the ocean surface usually are accompanied by high rates of evaporation. Evaporation acts to increase the surface density in two ways - by energy loss from the surface through the latent energy needed to cause the evaporation; and by increasing the surface salinity as the pure water is evaporated away, leaving behind in the ocean surface layer a greater concentration of dissolved solids. Therefore, convective mixing can develop from both sensible heat loss and a salinity increase at the surface of the ocean.

The depth to which convective mixing extends will depend to a degree on the initial density gradient present in the column of mixed-ocean water. When the initial density gradient is small, a given amount of cooling or evaporation will set off convection which will reach a relatively great depth. When the initial density gradient is large, convective mixing will extend downward a relatively small amount. For given cooling amounts, a

certain critical density gradient can be determined which will not permit convective mixing to extend below a certain depth.

The density after mixing of several ocean layers is greater than the weighted-average density of the layers mixed if a salinity gradient is present, due to the non-linear dependence of density on temperature and salinity (pressure effects can be ignored when compared to the temperature and salinity effects on density in the first 100m of the ocean below the surface) [1]. This phenomenon has been called "contraction on mixing" by N. N. Zubov [2].

N. P. Bulgakov has proposed a model to determine the limiting depth of convective mixing for given initial conditions [3]. In this model he has attempted to account for this non-linear dependence of density on temperature and salinity, especially in brackish water areas in sub-polar and polar regions. The model gives a simple numerical or graphical determination of the maximum depth of convective mixing. According to this model the limiting depth of mixing without ice formation occurs when the layer has been cooled to its freezing temperature; any further increase in convective mixing might be caused by a salinity increase due to ice formation at the ocean surface. In the greatest part of the open oceans, the upper layer is not

cooled to its freezing temperature, and this limiting depth of convective mixing usually is not realized.

Bulgakov's equation (to be discussed in detail in the next section) is the basis for this study which applies the convective model to data from a particular ocean region during months when convection is anticipated as a dominant mixing process. The model gives estimated depths of the homogeneous layer which may be verified against measured depths. Resolution of consistent differences provides an estimate of the validity of the model as well as a suggestion of the relative importance of convective mixing in that ocean situation.

3. Bulgakov's Model.

In order to understand Bulgakov's approach to determination of the limiting depth of convective mixing, several definitions are needed.

1) Density values are in terms of sigma values where

$$\text{sigma } (\sigma) = (\text{density} - 1) \times 10^3.$$

2) Reduced density (σ_r) is the maximum density for any given value of salinity (at the freezing temperature if the salinity is greater than 24.695 ‰ or at the temperature of maximum density if the salinity is less than 24.695 ‰).

3) Convective mixing depth (H_c or H) which is the depth of the homogeneous layer after mixing due to convective processes (less than the depth to the bottom of the halocline).

4) A one-layer ocean is defined as one where the upper boundary of the halocline (thermocline) reaches the surface, with consequent increasing salinity (decreasing temperature) commencing at the surface and continuing to a finite depth.

A two-layer ocean is defined as one where the upper boundary of the halocline (thermocline) lies below a homogeneous mixed upper layer.

5) Zero level (subscript 0) which is the top of the halocline in a two-layer ocean; the ocean's surface in a one-layer ocean.

6) Actual density (σ_t) which is the density corresponding to the temperature and salinity at any given level.

Bulgakov's approach relates, for a vertical column of ocean extending through the halocline, the profiles of initial density distribution and average salinity after mixing (plotted against depth) in a basic equation [3]. Figures 1 and 2 are diagrams of the geometric representations for a one-layer and a two-layer ocean respectively. The initial salinity distribution is represented by the line labeled S; due to the corresponding σ_t scale, this line also represents the reduced density distribution, σ_r, prior to mixing. Assuming that salinity varies uniformly with depth, the line labeled $\bar{S}$ represents the mixed-layer salinity with changing depth of the convective layer as well as the reduced density of the layer after mixing; these values are plotted against the existing depth of convective mixing. The initial density distribution is represented by the line labeled σ_t. The important plots on these diagrams are the averaged-salinity curves and the initial density distribution curves. The limiting depth to which convective

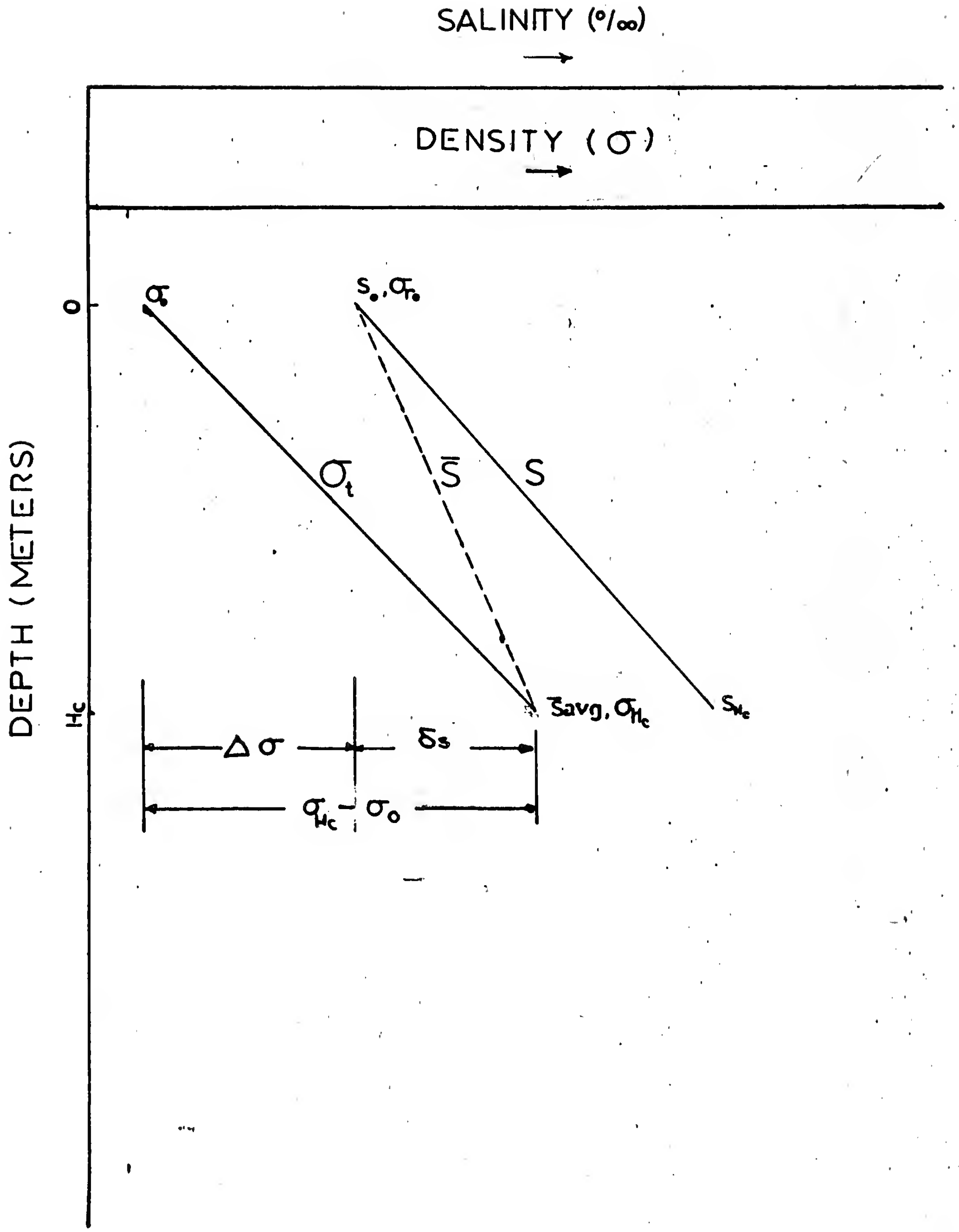

Fig. 1 - Determination of Depth of Convective Mixing for a One-layer Ocean.

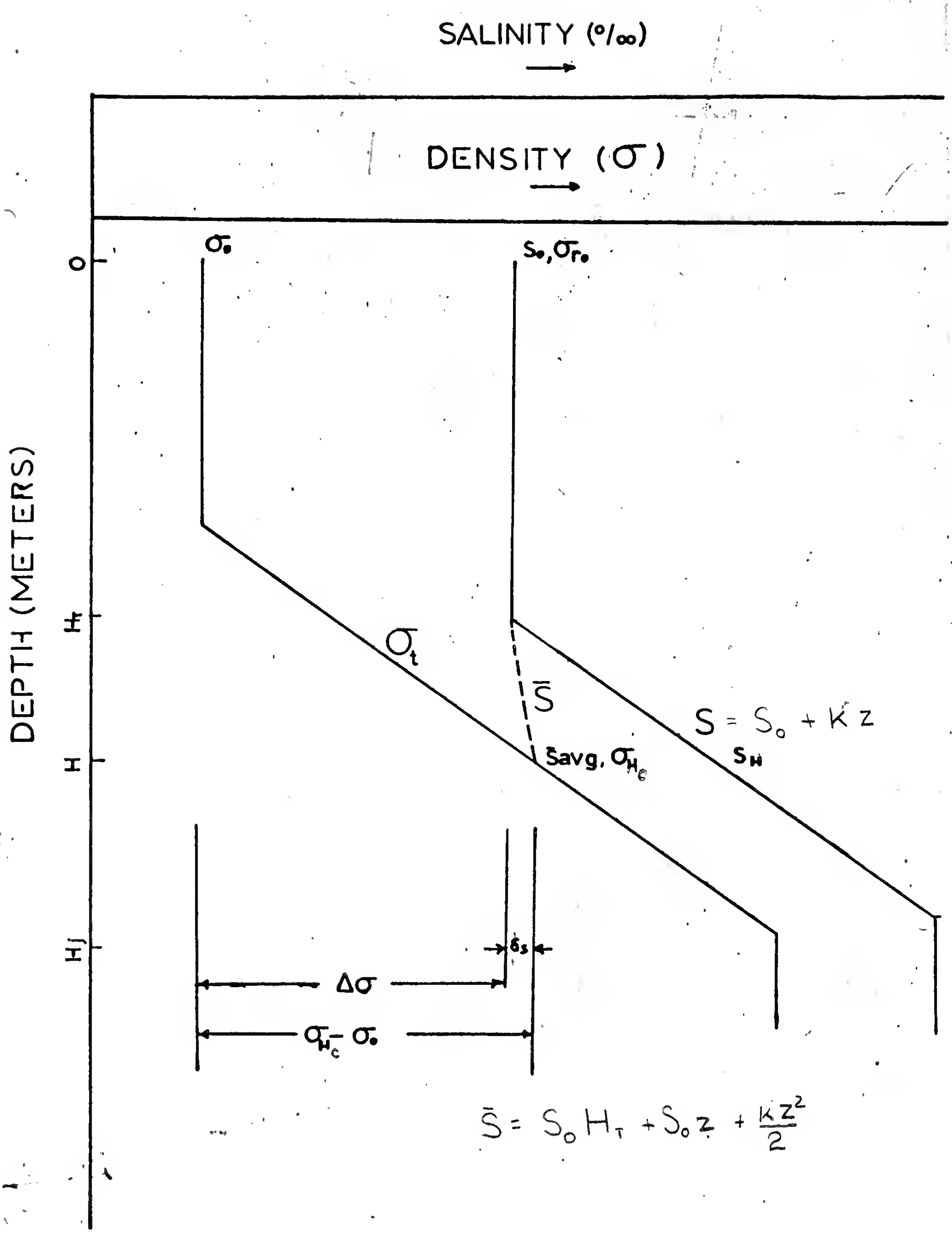

Fig. 2 - Determination of Depth of Convective Mixing for a Two-layer Ocean.

processes can occur is determined by the intersection of these two curves. It is at this point that the actual density (in the initial density distribution) is equivalent to the reduced density (or maximum density) of the homogeneous layer after convective mixing.

The geometrical construction of figures 1 and 2 provide a basis for the equation

$$\sigma_{H_c} - \sigma_o = \Delta\sigma + \delta s . \tag{1}$$

It should be noted that even though (1) is derived from the geometrical construction of the figures, each of the quantities has a physical interpretation. They are:

1) $\sigma_{H_c} - \sigma_o$ is the difference prior to mixing between the density at the depth of convective mixing, σ_{H_c}, and the density at the zero level;
2) $\Delta\sigma$ is the difference between the reduced density at the zero level, σ_{r_o}, and the density at the zero level, σ_o, (surface or top of the thermocline); this term represents the temperature reduction necessary in order to mix the layer down to its maximum depth;
3) δs is the difference between the reduced density at the zero level, σ_{r_o}, and the density at the depth of convective mixing, σ_{H_c}.

Derivation of the depth of convective mixing, H_c, for a one-layer ocean based upon equation (1) follows. In figure 1 it is assumed that the initial salinity distribution varies uniformly with depth; therefore, the final salinity of the mixed-layer is

$$\bar{S}_{avg} = \frac{S_{H_c} + S_o}{2} . \tag{2}$$

Equation (1) can be transformed to

$$\sigma_{H_c} - \sigma_o = \frac{d\sigma}{dz} H_c . \tag{3}$$

The quantity δs is reduced to

$$\delta s = 0.38 \frac{dS}{dz} H_c , \tag{4}$$

where $\frac{dS}{dz}$ is the salinity gradient in the halocline; 0.38 is the change in reduced density per corresponding change in salinity. The quantity $\frac{\delta s}{H_c}$ represents the slope of the dashed curve of figure 1. The constant, 0.38, is obtained by

$$\frac{\sigma_{H_c} - \sigma_{\tau_o}}{S_{H_c} - S_o} = 0.38 .$$ [1]

Equation (1) may now be written as

$$\frac{d\sigma}{dz} H_c = \Delta\sigma + 0.38 \frac{dS}{dz} H_c \tag{5}$$

[1] The constant 0.38 is determined by Bulgakov; this author, using Knudsen's Oceanographic Tables, determined the constant to be 0.40 for the salinity range of 32.0 to 34.0 .

from which the limiting depth of convective mixing, H_c, can be evaluated if the temperature at the surface (to evaluate $\Delta\sigma$), and the salinity and the density distribution are known.

The derivation of an equation analogous to equation (5) for a two-layer ocean resembles that for a one-layer ocean, with a few important differences as figure 2 illustrates. The derivation of a vertically-averaged salinity for the mixed-layer after convection is not as simple as in the one-layer case. Since Bulgakov does not give a detailed derivation for the two-layer ocean case, the following reasonable assumptions are made by this author in deriving the final equation stated by Bulgakov. The assumptions are

1) the salinity distribution of the upper mixed layer is uniform with depth,
2) the salinity varies uniformly with depth in the halocline, and
3) the curve of the average salinity when plotted against maximum depth of convective mixing is essentially parabolic in shape within the halocline.

Any weighted-average salinity for the convectively-mixed layer will give a much smaller value for the two-layer ocean case than for the one-layer case when convection

penetrates to a given depth into the halocline. The low-salinity water of the upper layer in the two-layer case acts to dilute the convectively-mixed layer.

It can be shown that the two-layer ocean counterpart of equation (4) becomes

$$\delta s = 0.19 \frac{dS}{dz} \frac{H^2}{H_j} , \qquad (6)$$

where H_j is the depth to the bottom of the halocline and $\frac{dS}{dz}$ is the initial salinity distribution with depth in the halocline. The equation comparable to (5) then becomes

$$\frac{d\sigma}{dz} H = \Delta\sigma + 0.19 \frac{dS}{dz} \frac{H^2}{H_j} . \qquad (7)$$

Equation (7) may be solved then for H, which represents the maximum change in depth of a homogeneous layer due to convective processes when all sensible heat above the freezing temperature has been removed from this layer. Equation (7) is the key equation in this study of convection penetration into the halocline; (7) will be referred to hereafter as either Bulgakov's model or Bulgakov's equation.

Bulgakov's equation may be used as a tool for determining maximum changes in the depth to the top of the thermocline (assuming that the top of the thermocline corresponds to the top of the halocline). From equation (7) it is seen that H, the change in depth due to convection, depends on obtaining

1) the density and salinity gradients in the halocline,

2) the actual and the reduced density at the top of the halocline, and

3) the depth to the bottom of the halocline.

The salinity gradients can be determined either from recent oceanographic stations or from climatology of the region. Actual density gradients may be approximated by using actual temperatures at the top and bottom of the halocline from bathythermograph (BT) observations and averaged salinities. Reduced density is evaluated initially as a function of salinity (which determines the freezing temperature)[1] from tables; once determined, the reduced density scale may be plotted as an alternate scale for the salinity scale as in figures 1 and 2; thus the plot of salinity versus depth may be read also as the plot of reduced density versus depth.

Thus equation (7) can be used readily to compute the maximum H (due to convection processes) for any area of the ocean where vertical salinity and temperature structure is known.

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[1]The temperature of maximum density was the freezing temperature due to the high salinity of the test area. Had the salinity been less than 24.695‰, the temperature of maximum density would have been used.

4. Application.

The northeastern Pacific Ocean was the area chosen to test the usefulness of Bulgakov's equation for estimating the convective change in the depth of the mixed layer as described by the temperature structure (thermocline) in the region. The data were taken by the Canadian weather ships which occupy the ocean weather station "P", located at 50° north latitude and 145° west longitude.

The area has two distinct features which make it ideal for study. First, the station is located near the center of the Alaskan current gyral. Thus the advective change of vertical temperature structure due to permanent currents can be considered negligible. Secondly, the area is characterized by a permanent halocline, located at a depth of about 100m at ship "P". The halocline separates the seasonally-affected upper layer from the nearly constant features of the lower layer [4]. The upper mixed layer has an annual average salinity of 32.7 ‰, while the average salinity at the bottom of the halocline is 33.8 ‰ [5].

The density structure of the area was considered to be a function of the temperature and salinity only. The adiabatic changes were considered negligible as were the effects due to pressure change with depth, in accordance with Sverdrup's analysis of stability within the upper 100m of the ocean [1].

For a more detailed analysis of the physical structure of the northeastern Pacific, the reader is referred to Dodimead, Favorite and Hirano[5].

The data supplied by the Pacific Oceanographic Group of Canada included both oceanographic station data and BT observations at ocean station "P" [7]. The data are considered to be extremely reliable. Eight oceanographic stations are taken (on the average) each month; and at least two BT observations are taken daily at 0200z and 1700z hours.

The data used were from November, 1957, through March, 1962, for the months of November through March. Mean salinity gradients for each month were determined using the oceanographic stations (See values in Table I). The density gradients were determined for each observation using actual temperatures at the top and the bottom of the thermocline from BT observations and salinities of 32.7 and 33.8 ‰ respectively. At the top of the thermocline the reduced density is 26.245 gm-cm^{-3}. All densities are given as sigma values, where

$$\text{sigma} = (\text{density} - 1) \times 10^3.$$

TABLE I - Mean Salinity Gradients

Month	$\frac{dS}{dz}$ ($\frac{‰}{m}$)
November	.0144
December	.0162
January	.0134
February	.0268
March	.0157

The Control Data Corporation 1604 Computer was used along with the observed data mentioned above to solve equation (7) for the change in depth to the top of the thermocline, H, attributed to convective mixing. The density gradient, $\frac{d\sigma}{dz}$, reduced density, σ_r, and actual density, σ_t, were evaluated from the observed data while the salinity gradient, $\frac{dS}{dz}$, was taken from the averaged data. After obtaining the quantity, H, this value was then added to the depth of the top of the thermocline to obtain the predicted depth of the mixed layer.

As a second application of equation (7), the computer was used to solve for the density gradient using arbitrary values of H (1m increments), averaged values for $\Delta\sigma$ and $\frac{dS}{dz}$, and arbitrary values for H_j. An analysis of various terms of equation (7) rewritten as

$$\frac{d\sigma}{dz} = \underset{(1)}{\frac{\Delta\sigma}{H}} + 0.19 \underset{(2)}{\frac{dS}{dz}} \frac{H}{H_j} \tag{7a}$$

shows that term (1) predominates in determining $\frac{d\sigma}{dz}$, since the vertical salinity gradients are small in the open oceans as is the ratio $\frac{H}{H_j}$ in term (2). At ship "P" term (2) is negligible in comparison with term (1) for the computation of $\frac{d\sigma}{dz}$, and equation (7a) may be written as

$$\frac{d\sigma}{dz} = \frac{\Delta\sigma}{H} \tag{7b}$$

Table II is based upon equation (7b) using 1m increments of H and averaged values of $\Delta\sigma$ (values are given in the table). This table, therefore, may be used to determine H by entering with a computed value of $\frac{d\sigma}{dz}$. The value of $\frac{d\sigma}{dz}$ may be computed from a current BT observation as outlined previously for application with equation (7). The interpretation which must be placed on the quantity $\frac{d\sigma}{dz}$ is that it represents the critical value of the density gradient; at density gradient values greater than $\frac{d\sigma}{dz}$, the convective processes will terminate at a depth equal to or less than the determined value of H.

TABLE II - DENSITY GRADIENT VS CONVECTIVE DEPTH CHANGE

NOVEMBER		DECEMBER		JANUARY		FEBRUARY		MARCH	
$\frac{d\sigma}{dz}$	H	$\frac{d\sigma}{dz}$	H	$\frac{d\sigma}{dz}$	H	$\frac{d\sigma}{dz}$	H	$\frac{d\sigma}{dz}$	H
$\Delta\sigma$ = .961		$\Delta\sigma$ = .545		$\Delta\sigma$ = .509		$\Delta\sigma$ = .487		$\Delta\sigma$ = .489	
.961	1.0	.545	1.0	.509	1.0	.487	1.0	.489	1.0
.480	2.0	.272	2.0	.254	2.0	.243	2.0	.244	2.0
.320	3.0	.182	3.0	.170	3.0	.162	3.0	.163	3.0
.240	4.0	.136	4.0	.127	4.0	.122	4.0	.122	4.0
.192	5.0	.109	5.0	.102	5.0	.097	5.0	.098	5.0
.160	6.0	.091	6.0	.085	6.0	.081	6.0	.081	6.0
.137	7.0	.078	7.0	.073	7.0	.070	7.0	.070	7.0
.120	8.0	.068	8.0	.064	8.0	.061	8.0	.061	8.0
.107	9.0	.061	9.0	.057	9.0	.054	9.0	.054	9.0
.096	10.0	.054	10.0	.051	10.0	.049	10.0	.049	10.0
.087	11.0	.050	11.0	.046	11.0	.044	11.0	.044	11.0
.080	12.0	.045	12.0	.042	12.0	.041	12.0	.041	12.0
.074	13.0	.042	13.0	.039	13.0	.037	13.0	.038	13.0
.069	14.0	.039	14.0	.036	14.0	.035	14.0	.035	14.0
.064	15.0	.036	15.0	.034	15.0	.032	15.0	.033	15.0
.060	16.0	.034	16.0	.032	16.0	.030	16.0	.031	16.0
.057	17.0	.032	17.0	.030	17.0	.029	17.0	.029	17.0
.053	18.0	.030	18.0	.028	18.0	.027	18.0	.027	18.0
.051	19.0	.029	19.0	.027	19.0	.026	19.0	.026	19.0
.048	20.0	.027	20.0	.025	20.0	.024	20.0	.024	20.0
.046	21.0	.026	21.0	.024	21.0	.023	21.0	.023	21.0
.044	22.0	.025	22.0	.023	22.0	.022	22.0	.022	22.0
.042	23.0	.024	23.0	.022	23.0	.021	23.0	.021	23.0
.040	24.0	.023	24.0	.021	24.0	.020	24.0	.020	24.0
.038	25.0	.022	25.0	.020	25.0	.019	25.0	.020	25.0
.037	26.0	.021	26.0	.020	26.0	.019	26.0	.019	26.0
.036	27.0	.020	27.0	.019	27.0	.018	27.0	.018	27.0
.034	28.0	.019	28.0	.018	28.0	.017	28.0	.017	28.0
.033	29.0	.019	29.0	.018	29.0	.017	29.0	.017	29.0
.032	30.0	.018	30.0	.017	30.0	.016	30.0	.016	30.0
.031	31.0	.018	31.0	.016	31.0	.016	31.0	.016	31.0
.030	32.0	.017	32.0	.016	32.0	.015	32.0	.015	32.0
.029	33.0	.017	33.0	.015	33.0	.015	33.0	.015	33.0
.028	34.0	.016	34.0	.015	34.0	.014	34.0	.014	34.0
.027	35.0	.016	35.0	.015	35.0	.014	35.0	.014	35.0
.027	36.0	.015	36.0	.014	36.0	.014	36.0	.014	36.0
.026	37.0	.015	37.0	.014	37.0	.013	37.0	.013	37.0
.025	38.0	.014	38.0	.013	38.0	.013	38.0	.013	38.0
.025	39.0	.014	39.0	.013	39.0	.012	39.0	.013	39.0
.024	40.0	.014	40.0	.013	40.0	.012	40.0	.012	40.0
.023	41.0	.013	41.0	.012	41.0	.012	41.0	.012	41.0
.023	42.0	.013	42.0	.012	42.0	.012	42.0	.012	42.0
.022	43.0	.013	43.0	.012	43.0	.011	43.0	.011	43.0
.022	44.0	.012	44.0	.012	44.0	.011	44.0	.011	44.0
.021	45.0	.012	45.0	.011	45.0	.011	45.0	.011	45.0
.021	46.0	.012	46.0	.011	46.0	.011	46.0	.011	46.0
.020	47.0	.012	47.0	.011	47.0	.010	47.0	.010	47.0
.020	48.0	.011	48.0	.011	48.0	.010	48.0	.010	48.0
.020	49.0	.011	49.0	.010	49.0	.010	49.0	.010	49.0

5. Interpretation of Data.

Interpretation of data is highly subjective due mainly to the defining of a thermocline. This author interpreted the top of a thermocline as requiring at least a 0.5C decrease of temperature in 10m. The bottom was interpreted as the location where the temperature decreased less than 0.5C in 10m.

A decision had to be made as to when only convective mixing is assumed to have occurred. Since the surface salinity changes at this station are very small, it can be assumed that evaporation effects on density were negligible. Convective mixing is assumed, thus, as a function only of heat loss in the surface layers during these cooling season months. From BT observations, it is extremely difficult to read the sea surface temperature to an accuracy greater than 0.1C. Therefore, it was assumed that when the sea surface temperature decreased by 0.1C or more in any period of time, only convective mixing took place.

6. Determination and Evaluation of Results.

Table III is a condensation of the statistical analysis of predicted mixed-layer depth in relation to observed mixed-layer depth. Approximately 1,100 BT observations were taken during the period of time covered in the analysis. Only 133 of these observations were used, in the final analysis, as representative of purely convective changes. Bulgakov's equation for determining H in a two-layer ocean was used to derive the predicted mixed-layer depth. The predicted value was arrived at by adding to the mixed-layer depth at the time of prediction the computed depth change due to convection.

Without exception, the arithmetic mean of the predicted mixed-layer depth, $\overline{MLD}_1$, was greater than the mean observed mixed-layer depth, $\overline{MLD}$. Equation (7) predicts a maximum change associated with the removal from the layer of all sensible heat above freezing temperature; since this seldom occurs, a correction factor was developed to correct this over-prediction of convective mixing influence at the top of the thermocline.

The over-prediction is not linear with temperature decrease in the surface layer which prevents the use of a constant correction factor. However, since the over-prediction is related to the retention of sensible heat, a ratio of change in sensible heat to initial total sensible heat was

TABLE III - Statistical Analysis of Data Results

	November	December	January	February	March
r_1	.323	.666	.678	.871	.926
r_2	.872	.668	.698	.876	.940
$\overline{MLD}$	68	100	108	121	124
$\overline{MLD}_1$	82	105	113	128	132
$\overline{MLD}_2$	68	101	106	120	122
SD	11.8	12.1	6.2	7.4	11.2
SD_1	9.1	14.8	7.8	7.5	11.2
SD_2	9.8	11.5	6.2	7.5	10.2
IW	9.0	12.0	12.0	14.0	15.0

r_1 - the linear correlation of predicted MLD to observed MLD

r_2 - the linear correlation of the corrected prediction of MLD to observed MLD (meters)

$\overline{MLD}$ - arithmetic mean of observed MLD (meters)

MLD_1 - arithmetic mean of predicted MLD (meters)

MLD_2 - arithmetic mean of corrected MLD (meters)

SD - standard deviation from $\overline{MLD}$ (meters)

SD_1 - standard deviation from $\overline{MLD}_1$ (meters)

SD_2 - standard deviation from $\overline{MLD}_2$ (meters)

IW - amplitude of internal waves at the mean observed $\overline{MLD}$ for waves of a period of 7 hours (meters)

used to multiply the initially computed value of H. The corrected change in depth due to convective mixing is

$$H_{corrected} = \frac{(T_{max} - T_o)}{T_{max}} H_{computed} . \qquad (8)$$

T_{max} is the maximum annual temperature of the ocean surface and T_o is the temperature of the ocean surface at time of prediction. A climatological mean T_{max} of 13.4C was used in the correction factor [4]. Thus the correction factor is a function of T_o only. A correction factor for each T_o has been computed and is displayed in Table IV.

The arithmetic mean of the corrected mixed-layer depth, $\overline{MLD_2}$, was in close agreement with the observed mean mixed-layer depth, $\overline{MLD}$.

The linear correlation of predicted to observed mixed-layer depth both before and after correction are given in Table III. With the exception of November, the correlation coefficients increased as the cooling season progressed. In November, the great difference in the correlation coefficients before and after correction appears to involve the high temperatures of the surface waters during the early part of the month. An analysis of October was dropped because heating season effects were predominantly superimposed upon the convective or cooling season effects; the mixed-layer depth in October was too shallow to escape the frictional mixing of wind-induced turbulence.

TABLE IV - Initial Temperature vs. Corrective Factor

Initial Temperature, To (C)	Correction Factor	Initial Temperature To (C)	Correction Factor
4.1	.694	8.2	.388
4.2	.686	8.3	.380
4.3	.679	8.4	.373
4.4	.672	8.5	.366
4.5	.664	8.6	.358
4.6	.657	8.7	.351
4.7	.649	8.8	.343
4.8	.642	8.9	.336
4.9	.634	9.0	.328
5.0	.627	9.1	.321
5.1	.619	9.2	.313
5.2	.612	9.3	.306
5.3	.604	9.4	.298
5.4	.597	9.5	.291
5.5	.590	9.6	.284
5.6	.582	9.7	.276
5.7	.575	9.8	.269
5.8	.567	9.9	.261
5.9	.560	10.0	.254
6.0	.552	10.1	.246
6.1	.545	10.2	.239
6.2	.537	10.3	.231
6.3	.530	10.4	.224
6.4	.522	10.5	.216
6.5	.515	10.6	.209
6.6	.507	10.7	.201
6.7	.500	10.8	.194
6.8	.492	10.9	.187
6.9	.485	11.0	.179
7.0	.478	11.1	.172
7.1	.470	11.2	.164
7.2	.463	11.3	.157
7.3	.455	11.4	.149
7.4	.448	11.5	.142
7.5	.440	11.6	.137
7.6	.433	11.7	.127
7.7	.425	11.8	.119
7.8	.418	11.9	.111
7.9	.410	12.0	.104
8.0	.403	12.1	.097
8.1	.396	12.2	.089

The standard deviations were derived for the computed values of MLD_1 and MLD_2 to determine if they are of the same order of magnitude as the observed MLD; in Table III, it can be seen that the order of magnitudes are related with a maximum difference of 2.7m.

The computed mixed-layer depth values, both uncorrected (MLD_1) and corrected (MLD_2), are shown by months and compared to observed values of mixed-layer depth (MLD) in figs. 3-7. The wide scatter of points in the figures tends to decrease as the cooling season progresses. Internal waves, it is believed, are the primary cause for the scatter of these data points. The amplitudes of the internal waves measured in the northeastern Atlantic by Glinskii and Boguslavskii range from 8m to 30m over a period range of 7.0 hours to 68.0 hours; the amplitude of waves is a maximum at a depth range of 100-200m. As an example, the amplitudes at 80m for periods of 24.8, 7.0, 14.8, 12.4, and 68.0 hours are approximately 8.0, 90, 9.5, 13 and 22m respectively[7]. Assuming that these results can be applied to a similar region in the Pacific, the standard deviations for MLD, MLD_1, and MLD_2 in Table III match closely the amplitudes of the 7-hour and the 12.4-hour internal waves

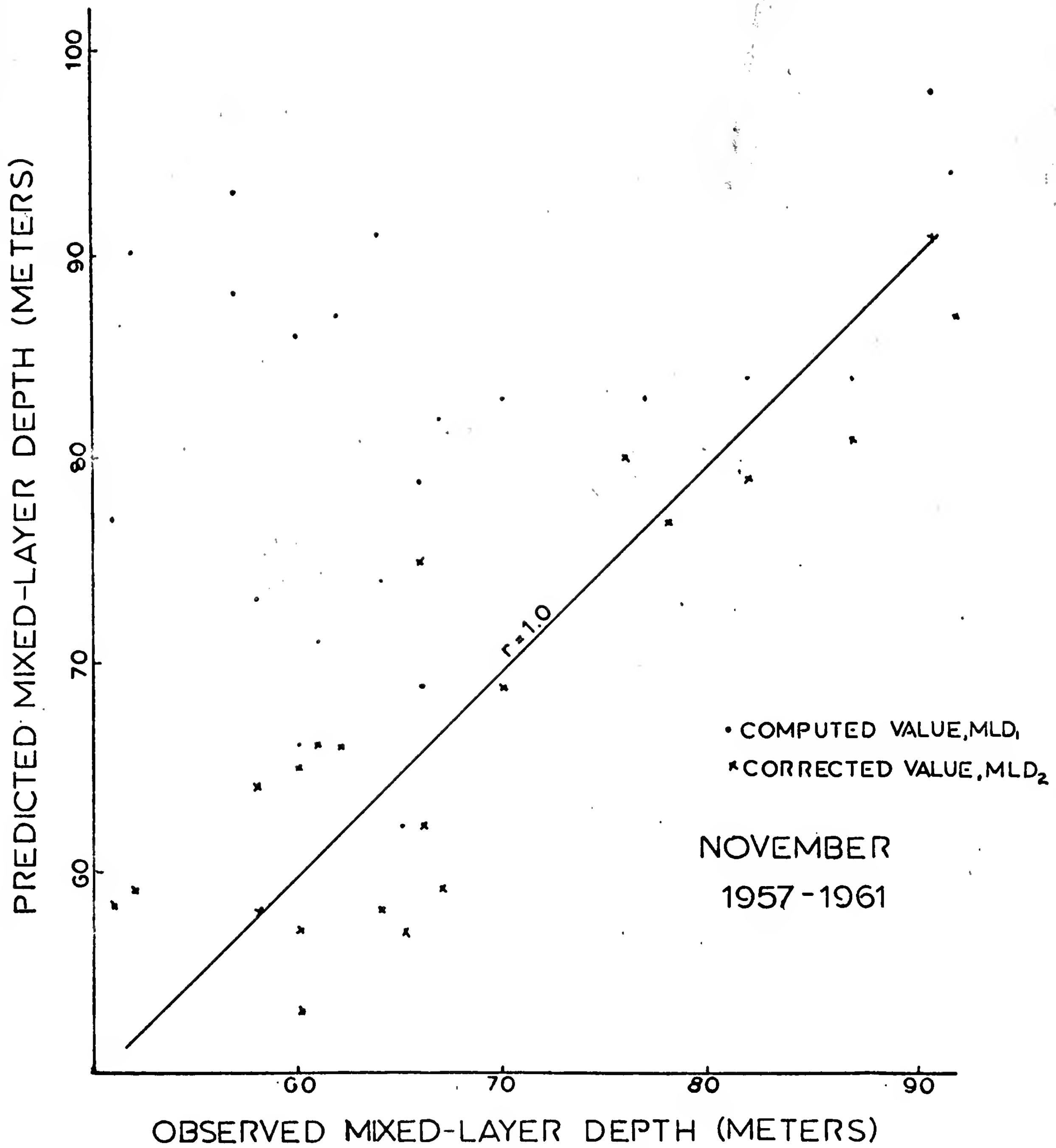

Fig. 3 - Observed vs. predicted mixed-layer depth for November

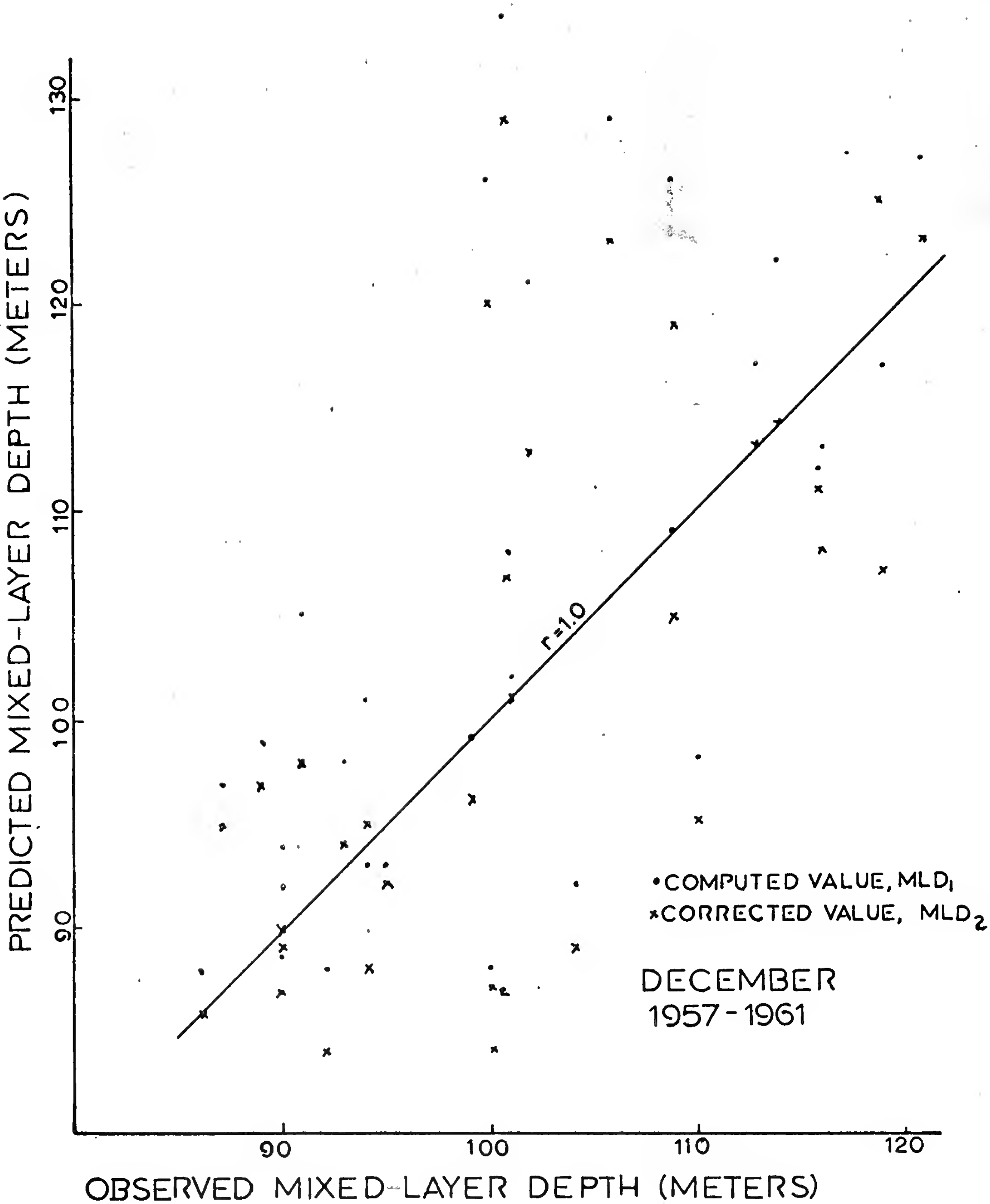

Fig. 4 - Observed vs. predicted mixed-layer depth for December

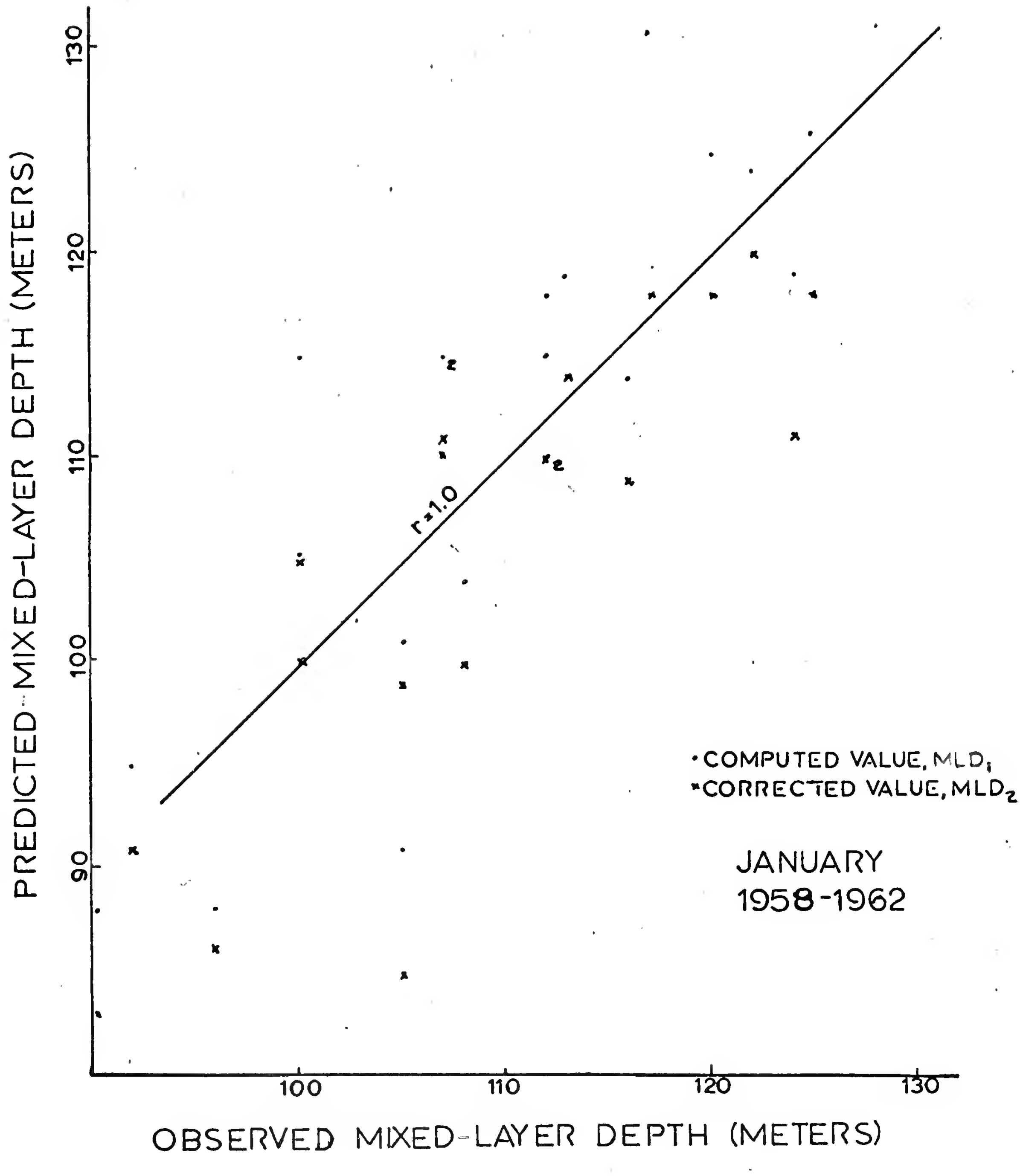

Fig 5. - Observed vs. predicted mixed-layer depth for January

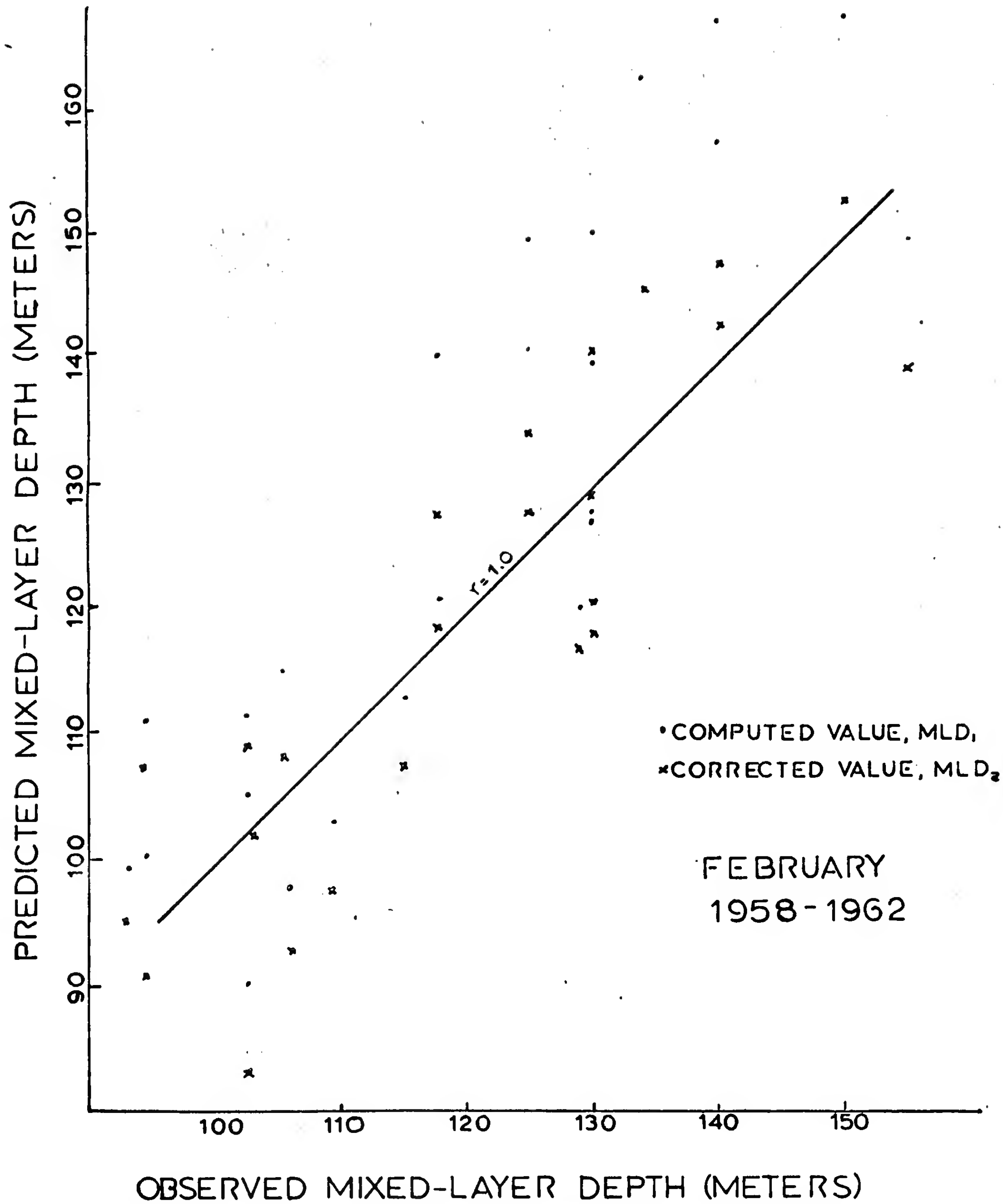

Fig 6 - Observed vs. predicted mixed-layer depth for February

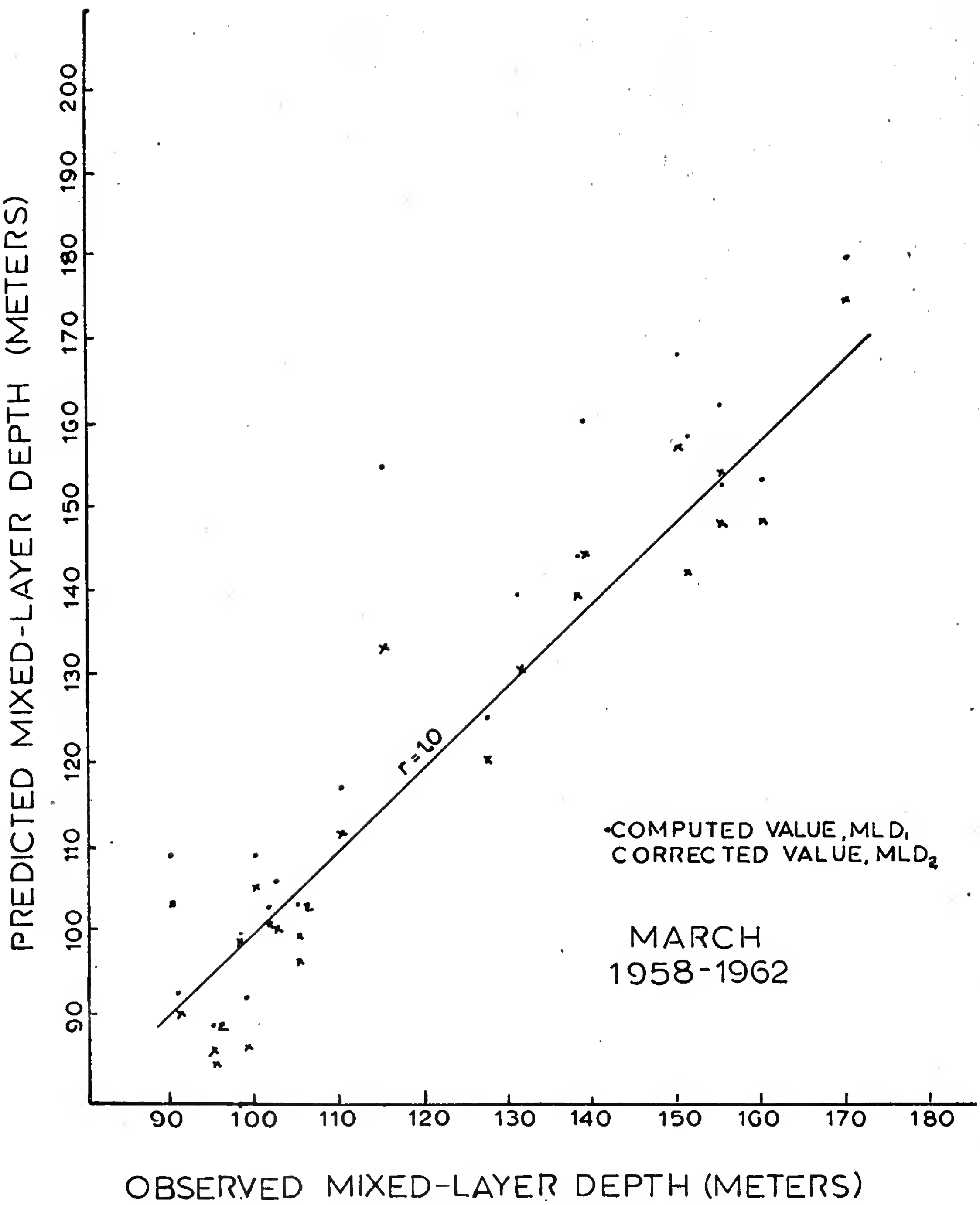

Fig. 7 - Observed vs. predicted mixed-layer depth for March

7. Conclusions and Acknowledgements.

It is apparent from the data results that Bulgakov's model can be useful in determining the maximum or limiting depth of convective mixing.

The main value of his work is that it gives a suggestion, in terms of easily-measured parameters, of the mechanics of convective activity in the open ocean. Further study might be attempted to separate the effects of convective mixing and frictional mixing using Bulgakov's equation as a basic tool. Present work in the heating season tends to emphasize the wind-induced frictional mixing while ignoring the increased density (thus convective mixing) effects of mixing two or more layers of the oceans. Contraction on mixing of several layers should be accounted for in the heating season and Bulgakov's equation might be used since contraction is automatically accounted for.

It can be concluded that Bulgakov's equation is useful in evaluating processes affecting the thermal structure of the oceans.

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